



Color fluctuation effects in proton–nucleus collisions

M. Alvioli^{a,b}, M. Strikman^{c,*}

^a ECT*, European Centre for Theoretical Studies in Nuclear Physics and Related Areas, Strada delle Tabarelle 286, I-38123 Villazzano (TN), Italy

^b CNR IRPI, via Madonna Alta 126, 06128 Perugia, Italy

^c 104 Davey Lab, The Pennsylvania State University, University Park, PA 16803, USA

ARTICLE INFO

Article history:

Received 12 January 2013

Received in revised form 22 March 2013

Accepted 22 April 2013

Available online 24 April 2013

Editor: J.-P. Blaizot

ABSTRACT

Color fluctuations in hadron–hadron collisions are responsible for the presence of inelastic diffraction and lead to distinctive differences between the Gribov picture of high energy scattering and the low energy Glauber picture. We find that color fluctuations give a larger contribution to the fluctuations of the number of wounded nucleons than the fluctuations of the number of nucleons at a given impact parameter. The two contributions for the impact parameter averaged fluctuations are comparable. As a result, standard procedures for selecting peripheral (central) collisions lead to selection of configurations in the projectile which interact with smaller (larger) than average strength. We suggest that studies of pA collisions with a hard trigger may allow to observe effects of color fluctuations.

© 2013 Elsevier B.V. All rights reserved.

1. Introduction

Currently most of the experimental studies as well as modeling of the nucleus–nucleus (proton–nucleus) collisions involve using the Glauber model. Namely, the number of involved nucleons is calculated probabilistically assuming that each Nucleon–Nucleon (NN) inelastic collision is determined by the value of σ_{in}^{NN} at the collision energy.

However, the dominance of large longitudinal distances in high energy scattering [1] changes qualitatively the pattern of multiple interactions. Indeed, in the Glauber approximation high energy interactions of the projectile with a target occur via consecutive rescatterings of the projectile off the constituents of the target. The projectile during the interactions is on mass shell – one takes the residues in the propagators of the projectile. This approximation contradicts the QCD based space–time evolution of high energy processes dominated by particles production. The projectile interacts with the target in frozen configurations since the life time of the configurations becomes much larger than the size of the target. Hence there is no time for a frozen configuration in the projectile to combine back into the projectile during the time of the order R_T , the radius of the target. As a result the amplitudes described by Glauber model diagrams die out at large energies $\propto 1/s$ (a formal proof which is based on the analytic properties of the Feynman diagrams was given in [2,3]).

In the Glauber model the number of interacting nucleons is calculated probabilistically assuming that the probability of individual

NN inelastic collisions is determined by the value of σ_{in}^{NN} at the collision energy. Fluctuations of the number of wounded nucleons at a given impact parameter are generated solely by fluctuations of the positions of nucleons in the nucleus and (in some models) due to peripheral collisions of nucleons, where the interaction is gray and hence the chance to interact differs from one or zero. Hard collisions are treated as binary collisions, which is equivalent to taking the diagonal generalized parton densities of nuclei, $f_A(x, Q^2, b)$, proportional to the impact factor $T(b)$:

$$F_A(x, Q^2, b) = f_N(x, Q^2)T(b), \quad (1)$$

where $T(b)$ is normalized as $\int db T(b) = A$. A nuclear shadowing correction is introduced for $x \leq 0.01$.

The high energy theory of soft interactions with nuclei was developed by Gribov [4] who expressed the shadowing contribution to the cross section of hadron–nucleus (hA) interactions through the contribution of non-planar diagrams. The Gribov–Glauber theory, in difference from the low energy Glauber theory, requires taking into account that a particular quark–gluon configuration of the projectile is frozen during the collision and that it may interact with different strength as compared to the average strength. This leads to fluctuations of the number of collisions which are significantly larger than in the Glauber model. The fluctuations of the strength of the interaction are related to the ratio of inelastic and elastic diffraction in NN scattering at $t = 0$. Relevance of fluctuations of the strength was first pointed out in [5,6] but these effects were never analyzed in detail before.

Another effect contributing to fluctuations of observables in hA collisions is fluctuations of the gluon density which can originate both from the fluctuations of the nucleon configurations and from

* Corresponding author.

E-mail address: strikman@phys.psu.edu (M. Strikman).

the fluctuations of the gluon densities in the individual nucleons. We will consider this effect elsewhere.

The Letter is organized as follows. In Section 2 we summarize the necessary information about fluctuations of the strength of NN interaction. In Section 3 we use the Gribov–Glauber model in the optical approximation to obtain analytic results for the strength of fluctuations of the number of wounded nucleons and relative contributions to these fluctuations of the fluctuations of the strength of the interaction and of geometry of collisions. In Section 4 we develop a full Monte Carlo (MC) model in which the geometry of projectile–target nucleon interaction is accounted for, and the strength of the interaction fluctuates on an event-by-event basis. The results for the change of the distribution over the number of collisions and for the dependence of the average strength of the interaction on impact parameter are presented. Possibilities for observing color fluctuation effects in collisions with hard triggers are outlined.

2. Color fluctuation effects in proton–nucleus collisions

2.1. Gribov inelastic shadowing

It was demonstrated by Gribov [4] that the nuclear shadowing contribution to the total cross section of the hadron–deuteron scattering can be expressed through the diffraction cross section at $t = 0$. Operationally this amounts to the replacement in the Glauber formulæ of the elastic hN cross section at $t \sim 0$ by the sum of elastic and diffractive cross section at $t = 0$, leading to an enhancement of the multinucleon interactions. For heavier nuclei the Gribov formulæ involve the coupling of the projectile to $N > 2$ vacuum exchanges which has to be modeled.

The contribution of the double scattering to the total hadron–nucleon (hN) cross section is enhanced by a factor $1 + \omega_\sigma$, where

$$\omega_\sigma = \frac{d\sigma(hN \rightarrow XN)}{dt} \bigg/ \frac{d\sigma(hN \rightarrow hN)}{dt} \bigg|_{t=0}. \quad (2)$$

The relation between the double scattering cross section and the total diffraction cross section can be naturally understood in the Good and Walker formalism [7], which provided the effective realization of the Feinberg–Pomeranchuk picture [8] of the inelastic diffraction. In this formalism one introduces eigenstates of the scattering matrix diagonal in σ ; see Ref. [9] for a review. Configurations with different σ_i scatter without interference off two target nucleons contributing in the case of scattering of two nucleons with strength $\propto \sigma_i^2$ to the shadowing of the total cross section. This is the same quantity as in the expression for the total cross section of hadron–nucleon diffraction at $t = 0$. This interpretation of the Gribov result for the shadowing correction to the total cross section was first given by Kopeliovich and Lapidus [10].

2.2. Distribution over the strength of interaction

The fluctuations of strength of interaction arise naturally in QCD where the strength of interaction depends on the volume occupied by color. In particular, the presence of some small configurations leads to fluctuations interacting with a small cross section. So we will refer to these fluctuations as color fluctuations.

In order to describe the effect of color fluctuations for a variety of processes it is convenient to introduce the notion of distribution over the strength of interaction, $P_h(\sigma_{tot})$ – the probability for an incoming hadron to interact with total cross section σ_{tot} . The distribution $P_h(\sigma_{tot})$ satisfies two normalization sum rules:

$$\int d\sigma_{tot} P_h(\sigma_{tot}) = 1, \quad \int d\sigma_{tot} \sigma_{tot} P_h(\sigma_{tot}) = \sigma_{tot}^{hN}, \quad (3)$$

and the Miettinen–Pumplin relation [11]

$$\int d\sigma_{tot} [\sigma_{tot}^2 / (\sigma_{tot}^{hN})^2 - 1] P_h(\sigma_{tot}) = \omega_\sigma, \quad (4)$$

where σ_{tot}^{hN} is the free cross section. Experimentally, ω_σ first grows with energy then starts dropping at energies $\sqrt{s} \geq 100$ GeV. There are no direct measurements at the RHIC energy of 200 GeV, but an overall analysis indicates that it is of the order 0.25. The first LHC data seem to indicate that inelastic diffraction still constitutes a large fraction of the cross section – it is comparable to the elastic cross section, suggesting $\omega_\sigma \sim 0.2$ at those energies. It is difficult at the moment to ascribe error bars to these numbers. However, it is expected that the values of ω_σ corresponding to the LHC energies will be soon measured with a good precision.

It is worth emphasizing here that these seemingly small values of ω_σ correspond to very large fluctuations of the interaction strength. For example, if we consider a simple two component model (equivalent to the quasi-eikonal approximation), in which two components are present in the projectile wave function with equal probability and interact with strengths $\sigma_{tot}^{(1)}$ and $\sigma_{tot}^{(2)}$:

$$\sigma_{tot}^{(1)} = \sigma_{tot}^{hN} (1 - \sqrt{\omega}), \quad \sigma_{tot}^{(2)} = \sigma_{tot}^{hN} (1 + \sqrt{\omega}). \quad (5)$$

Thus for $\omega_\sigma = 0.25$, we have $\sigma_{tot}^{(1)}/\sigma_{tot}^{hN} = 0.5$, $\sigma_{tot}^{(2)}/\sigma_{tot}^{hN} = 1.5$ and hence $\sigma_{tot}^{(1)}/\sigma_{tot}^{(2)} = 3$.

3. Gribov–Glauber model predictions for fluctuations in the optical approximation

In order to illustrate the effects of the color fluctuations and their interplay with the fluctuations of the local nuclear density we first consider the optical approximation of the Glauber model where the radius of the NN interaction is neglected as compared to the distance between the nucleons.

Within this model the total inelastic hadron–nucleus cross section σ_{in}^{hA} can be written as follows:

$$\sigma_{in}^{hA} = \int d\mathbf{b} (1 - [1 - x(b)]^A) = \sum_{N=1}^A \frac{(-1)^{N+1} A!}{(A-N)! N!} \int d\mathbf{b} x(b)^N, \quad (6)$$

where $x(b) = \sigma_{in}^{hN} T(b)/A$ and normalization $\int d\mathbf{b} T(b) = A$.

Note that in Eq. (6) nucleon–nucleon correlations in the nuclear wave function are neglected as well as the finite radius of the hadron–nucleon interaction; an implementation of correlations in the optical limit in the Gribov–Glauber formalism can be found in Refs. [12,13] and in Ref. [14] within the MC approach and will not be discussed here. Eq. (6) can be rewritten as a sum of positive cross sections [15] as follows:

$$\sigma_{in}^{hA} = \sum_{N=1}^A \sigma_N, \quad \sigma_N = \frac{A!}{(A-N)! N!} \int d\mathbf{b} x(b)^N [1 - x(b)]^{A-N}, \quad (7)$$

where σ_N denotes the cross section of the physical process in which N nucleons have been involved in inelastic interactions with the projectile. Using Eq. (7), the average number of interactions $\langle N \rangle$ can be expressed as

$$\langle N \rangle = \sum_{N=1}^A N \sigma_N / \sum_{N=1}^A \sigma_N = \frac{\sigma_{in}^{hN}}{\sigma_{in}^{hA}} \int d\mathbf{b} T(b) = \frac{A \sigma_{in}^{hN}}{\sigma_{in}^{hA}}, \quad (8)$$

which coincides with the naive estimate of shadowing as being equal to the number of nucleons shadowed in a typical hA inelastic collision.

We can include color fluctuations by allowing the inelastic cross section σ_{in} to be distributed according to a proper distribution, $P_H(\sigma_{in})$:

$$\sigma_{in}^{hA} = \int d\sigma_{in} P_H(\sigma_{in}) \int d\mathbf{b} (1 - [1 - x(b)]^A), \quad (9)$$

where now $x(b) = \sigma_{in} T(b)/A$, and

$$\sigma_N = \int d\sigma_{in} P_H(\sigma_{in}) \frac{A!}{(A-N)!N!} \int d\mathbf{b} x(b)^N [1 - x(b)]^{A-N}. \quad (10)$$

The probability of collisions with exactly N inelastic interactions in both Glauber model and the color fluctuation approximation is simply $R_N = \sigma_N / \sigma_{in}^{hA}$.

Using the equations above we can for example calculate the average number of collisions which is given by the same equation as for the Glauber model (Eq. (8)), leading to a very small (a few %) change of average N , as shown in Table 1, since the inelastic corrections to σ_{in}^{hA} are small for a realistic $P_H(\sigma_{in})$; see Ref. [13] and references therein. The physical reason why the corrections are small is that, in a broad range of b , the interaction is close to the black limit for all essential values of σ_{in} , so only a small range of (large) b contributes to inelastic shadowing corrections. At the same time the color fluctuation effect is large for the variance of the distribution over the number of collisions. Eq. (10) leads to

$$\langle N(N-1) \rangle = A(A-1) \frac{\langle \sigma_{in}^2 \rangle}{\sigma_{in}^{hA}} \int d\mathbf{b} T^2(b), \quad (11)$$

and hence the variance is equal to

$$\omega_N \equiv \frac{\langle N^2 \rangle}{\langle N \rangle^2} - 1 = \frac{A(A-1)}{\langle N \rangle^2} \frac{\langle \sigma_{in}^2 \rangle}{\sigma_{in}^{hA}} \int d\mathbf{b} T^2(b) + \frac{1}{\langle N \rangle} - 1. \quad (12)$$

One can see from Eq. (12) that the variance receives contributions both from the fluctuations of the impact parameter and from the fluctuations of σ_{in} . Using Eqs. (8), (11) we obtain for the variance in Eq. (12) the value of about 0.46 (RHIC) and 0.51 (LHC). Numerical values of the different terms in Eq. (12) are: $1.26 + 0.20 - 1 = 0.46$ (RHIC) and $1.38 + 0.13 - 1 = 0.51$ (LHC). The account of the color fluctuations practically does not change $\langle N \rangle$. It mainly changes the nominator of the first term by the factor $1 + \omega_\sigma$.¹ Though this change is rather small, the strong cancellation between the first and the third terms of Eq. (12) strongly enhances the effect of color fluctuations.

A more realistic treatment of the color fluctuations taking into account the profile function of the NN interactions and small effect of short-range correlations is possible in the MC model described in the next section. First, one calculates the probability $P_N(b)$ shown in Fig. 1 of having exactly N inelastic interactions at a given impact parameter b . Next one can calculate the quantity in Eq. (12) by integrating $P_N(b)$ over the impact parameter: $P_N = 2\pi \int b db P_N(b)$. The results are given in Table 1.

A comparison of some of the predictions of the optical approximation of the Glauber model and the MC calculations, which take into account finite radius of the NN interaction neglected in the optical model, will be given below.

4. Monte Carlo algorithm for modeling effects of fluctuations

We have seen from the analysis of the optical model that fluctuations in the number of wounded nucleons originate both from

Table 1

The fluctuations, as defined in Eq. (12), calculated both within the MC approach and optical model. We used no color fluctuation (Glauber), color fluctuations implemented with the two states model described in the text (GG2) and with the full color fluctuation model (GG $P_h(\sigma_{tot})$) described by the distribution $P_h(\sigma_{tot})$ of Eq. (16).

Energy/model	Monte Carlo			Optical model		
	$\langle N \rangle$	$\langle N^2 \rangle$	ω_N	$\langle N \rangle$	$\langle N^2 \rangle$	ω_N
RHIC, Glauber	4.6	31.6	0.51	5.0	35.9	0.46
RHIC, GG2	4.7	38.9	0.74	5.1	45.3	0.71
RHIC, GG $P_h(\sigma_{tot})$	4.8	39.2	0.72	5.2	45.6	0.70
LHC, Glauber	6.7	72.4	0.59	7.6	88.0	0.51
LHC, GG2	6.8	84.2	0.80	7.8	106.2	0.75
LHC, GG $P_h(\sigma_{tot})$	6.8	82.1	0.77	7.8	106.4	0.74

color fluctuations and from fluctuations of the number of nucleons along the path of the projectile.

The event-by-event fluctuations of the number of wounded nucleons due to the fluctuations in the number of nucleons at a given impact parameter are present already on the level of the Glauber model [14]. In the case when no fluctuations of σ are present, $\langle N(\sigma_{in}^{hN}) \rangle$ is given by Eq. (8). In this case we can write

$$\langle N(\sigma_{in}^{hN})^2 \rangle = \langle N \rangle^2 (1 + \omega_\rho(\sigma_{in}^{hN})), \quad (13)$$

where $\omega_\rho(\sigma_{in}^{hN})$ is the dispersion in the case of no color fluctuations. We found that $\omega_\rho(\sigma_{in}^{hN})$ drops as a function of σ_{in}^{hN} , as a consequence of the increasing number of nucleons in the interaction volume. In the calculations we use the event generator [14]. This event generator includes short-range correlations between nucleons, however this effect leads to a very small correction for the discussed quantity. The code also includes a realistic dependence of the probability of the NN interaction on the relative impact parameter of the projectile \mathbf{b} , and the target nucleon \mathbf{b}_j : $\mathbf{b} - \mathbf{b}_j$. The probability of the interaction is expressed through the impact factor of the NN elastic amplitude

$$\Gamma(\mathbf{b} - \mathbf{b}_j) = \frac{\sigma_{tot}^{hN}}{4\pi B} e^{-(\mathbf{b} - \mathbf{b}_j)^2 / 2B} \quad (14)$$

as follows:

$$P(\mathbf{b}, \mathbf{b}_j) = 1 - [1 - \Gamma(\mathbf{b} - \mathbf{b}_j)]^2. \quad (15)$$

Here we used the exponential fit to the elastic cross section $d\sigma/dt \propto \exp(Bt)$.

In order to perform numerical analyses we follow [16], and take the probability distribution for σ_{tot} as follows:

$$P_h(\sigma_{tot}) = \rho \frac{\sigma_{tot}}{\sigma_{tot} + \sigma_0} \exp \left\{ -\frac{(\sigma_{tot}/\sigma_0 - 1)^2}{\Omega^2} \right\}, \quad (16)$$

where ρ is a normalization constant and we have $\sigma_0 = 72.5$ mb and $\Omega = 1.01$ at LHC energies, while $\sigma_0 = 32.6$ mb and $\Omega = 1.49$ at RHIC energies. One can verify that the distribution of Eq. (16) satisfies the sum rules (3), (4), with our values $\sigma_{tot}^{hN} = \sigma_{tot}^{NN} = 51.95$ mb for RHIC and $\sigma_{tot}^{hN} = \sigma_{tot}^{NN} = 94.8$ mb for LHC energies.

When converting from the distribution over σ_{tot} , $P_h(\sigma_{tot})$, to the distribution over σ_{in} , $P_H(\sigma_{in})$, we used the geometric scaling observation that the t -slope of the elastic scattering is proportional to σ_{tot} . So the ratio $\sigma_{in}/\sigma_{tot} = \lambda$ weakly depends on the projectile and energy. Hence we take $\lambda = \text{const}$, so that we simply have to use a Jacobian $1/\lambda$, with

$$P_H(\sigma_{in}) = P_h(\sigma_{tot})/\lambda, \quad \sigma_{in} = \lambda\sigma_{tot}. \quad (17)$$

Indeed in this case $\int d\sigma_{in} P_H(\sigma_{in}) = 1$ holds as well. This corresponds to $B(\sigma_{tot}) = B(\sigma_{tot}^{hN})\sigma_{tot}/\sigma_{tot}^{hN}$.

¹ We assume here that fluctuations for the inelastic and total cross sections are similar, cf. discussion before Eq. (17).

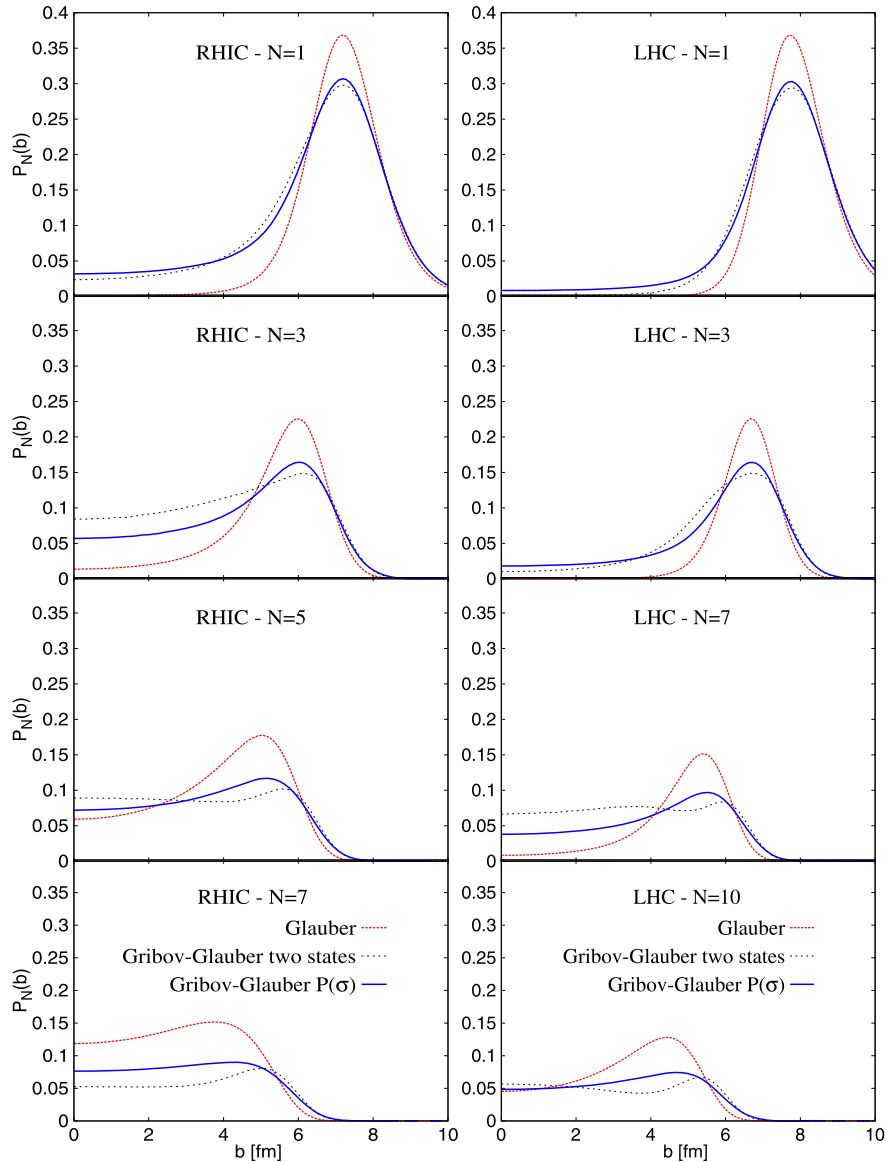


Fig. 1. The probability $P_N(b)$ of having N inelastically interacting (wounded) nucleons in a pA collision, vs. impact parameter b , when using simple Glauber (red curves), a two states model (black curves) and a distribution $P_h(\sigma_{tot})$ (blue curves); cf. Eq. (16). The $P_N(b)$'s are obtained by extension of the MC code of Ref. [14] to include color fluctuations. Top row shows $P_{N=1}(b)$; the remaining panels correspond to $N = \langle N \rangle$ and $N = \langle N \rangle \pm 0.5\langle N \rangle$. $\langle N \rangle$ is taken as 5 and 7 for RHIC and LHC energies, respectively (cf. Table 1). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

In our numerical studies we used the fluctuation distribution given by Eq. (16), σ_{tot}^{NN} given above and $B = 14 \text{ GeV}^{-2}$ (RHIC), $B = 19.38 \text{ GeV}^{-2}$ (LHC). This parametrization satisfies the s -channel unitarity condition $\Gamma(b) \leq 1$. In our model this condition holds automatically also for the elastic “color-fluctuation”-nucleon amplitude. Our algorithm is a natural extension of that of [14] – where distribution over N was calculated in the Glauber model neglecting effects of color fluctuations.

Since the contributions of states with different σ do not interfere, the probability $P_N(b)$ to have exactly N inelastic interactions at given b is²

$$P_N(b) = \int d\sigma_{tot} P_h(\sigma_{tot}) P_N(b; \sigma_{tot}), \quad (18)$$

² In this treatment we neglect small contributions of incoherent diffractive processes $pA \rightarrow XA^*$, which mostly contribute to $P_1(b)$.

where $P_N(b; \sigma_{tot})$ is calculated using the procedure of Ref. [14] for fixed σ_{tot}^{hN} in the Glauber model. Including color fluctuations results in a substantially broader distribution over b of the probability $P_N(b)$ of having exactly N interactions for a given impact parameter N , as shown in Fig. 1. The two component model gives the distributions pretty close to the distributions including full fluctuations. $P_N(b)$ are obviously normalized so that $\sum_N \int db P_N(b) = \sigma_{in}^{hA}$. The calculations of Table 1 have been performed integrating the quantities of Fig. 1 over the impact parameter: $P_N = \int db P_N(b)$; $\langle N \rangle = \sum_N N P_N / \sum_N P_N$; $\langle N^2 \rangle = \sum_N N^2 P_N / \sum_N P_N$.

Another quantity which characterizes the effects of spatial and color fluctuations is dispersion of the number of interactions at a given impact parameter, b . To illustrate the expected pattern let us first consider the case of small b and large A , when the probability of having at least one inelastic interaction is 1. In this case $\langle N \rangle = T(b)\sigma_{in}$, hence the dispersion of the distribution over N including both effects can be calculated as follows:

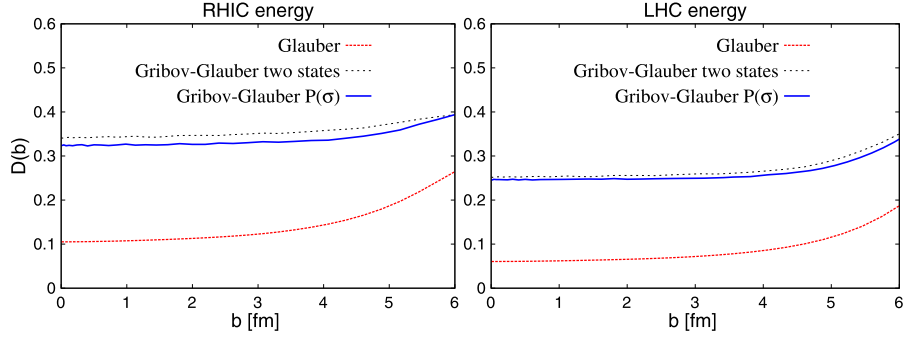


Fig. 2. Effect on fluctuations of the dispersion, Eq. (21), when using a distribution of σ_{tot} with two values of the cross section with equal probability and with $P_h(\sigma_{tot})$ given by Eq.(16), for realistic parameters corresponding to RHIC (left) and LHC (right) energies.

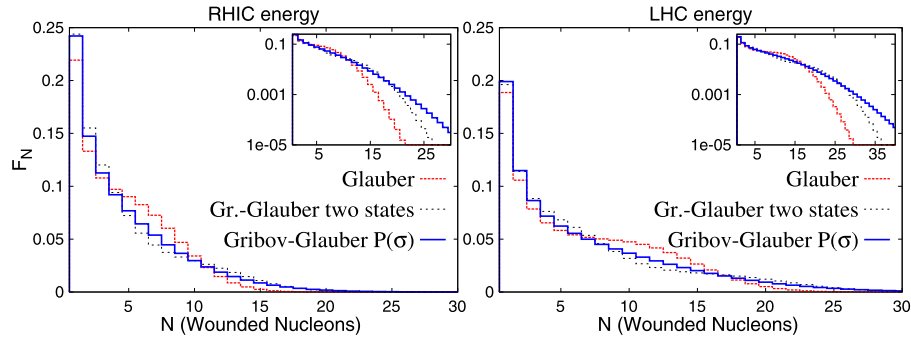


Fig. 3. Effect of the event-by-event fluctuating values of σ_{tot} , for RHIC (left panel) and LHC energies (right panel) on the number of wounded nucleons, calculated as $F_N = \int db P_N(b)/\sigma_{in}^{hA}$. Red curves show the results obtained with the usual Glauber calculation with fixed cross section, black curves correspond to calculations with the two component model and blue curves correspond to calculations with fluctuating cross section with $P_h(\sigma_{tot})$ distribution. The insets show the same quantities in logarithmic scale. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

$$\langle N^2 \rangle = \int d\sigma_{in} P_H(\sigma_{in}) \langle N \rangle^2 \left(\frac{\sigma_{in}}{\langle \sigma_{in} \rangle} \right)^2 (1 + \omega_\rho(\sigma_{in})). \quad (19)$$

Now we can calculate the total dispersion. The first term in $(1 + \omega_\rho)$ simply gives ω_σ . The second term takes into account the dependence of ω_ρ on the fluctuating σ_{in} :

$$\omega_{tot} = \omega_\sigma + \int d\sigma_{in} P_H(\sigma_{in}) \left(\frac{\sigma_{in}}{\langle \sigma_{in} \rangle} \right)^2 \omega_\rho(\sigma_{in}). \quad (20)$$

Since the integral in the second term is dominated by $\sigma_{in} > \sigma_{in}^{hN}$, for which ω_ρ is smaller than in correspondence of the average value of σ_{in} , ω_{tot} , Eq. (20) leads to a dispersion somewhat smaller than $\omega_\sigma + \omega_\rho(\sigma_{in}^{hN})$. This is consistent with the pattern we find in the numerical calculation presented in Fig. 2 for

$$D(b) = \frac{\langle N^2 \rangle_b - \langle N \rangle_b^2}{\langle N \rangle_b^2}, \quad (21)$$

$\langle N \rangle_b = \sum_N N P_N(b) / \sum_N P_N(b)$ and $\langle N^2 \rangle_b = \sum_N N^2 P_N(b) / \sum_N P_N(b)$. One can see that for RHIC and LHC energies the dominant effect comes from color fluctuations. Moreover, the two states approximation gives the result which is very close to the calculation with full $P_h(\sigma_{tot})$, so the two states model can be used to simplify modeling of color fluctuation effects.

The large variance of the distribution leads to a much wider distribution over N than in the Glauber model, as shown in Fig. 3. The figure shows the quantities $F_N = \int db P_N(b)/\sigma_{in}^{hA}$; the same quantities are plotted in logarithmic scale in the insets, and one can see that the color fluctuations produce a much stronger large N tail. Among other things, this implies that selection of events which in the Glauber model correspond to very central impact parameters actually gets a significant contribution from pretty large

impact parameters – for example, in the two component model discussed above the collisions at impact parameter b satisfying the condition $T(b)/T(0) = 1/(1 + \sqrt{\omega})$ with a probability of 1/2 generates the same number of wounded nucleons as average number of wounded nucleons at $b = 0$. For $\omega = 0.25$ we have $1/(1 + \sqrt{\omega}) = 0.67$ and this corresponds to $b \simeq 4.58$ fm.

An important implication of the broad distributions over N which is mostly due to fluctuations of the strength of the interaction is that selection of large N also selects configurations in the projectile nucleon with cross section larger than average. To illustrate this trend within our MC, let us consider the average σ_{tot} for events with a given number N of wounded nucleons. Denoting the probability to have exactly N wounded nucleons $P_N = \int db P_N(b)$ and using Eq. (18), we can write

$$\frac{\langle \sigma_{tot} \rangle_N}{\sigma_{tot}^{hN}} = \frac{1}{\sigma_{tot}^{hN}} \frac{\int d\sigma_{tot} db \sigma_{tot} P_h(\sigma_{tot}) P_N(b; \sigma_{tot})}{\int d\sigma_{tot} db P_h(\sigma_{tot}) P_N(b; \sigma_{tot})}. \quad (22)$$

The results of the calculation are presented in Fig. 4. One can see that selecting $N \gg \langle N \rangle$ leads to a significant enhancement of the contribution of configurations which have interaction strength larger than average. For small N average $\langle \sigma_{tot} \rangle_N$ is below σ_{tot}^{hN} , but the effect is relatively small especially for $N = 1$ where very peripheral collisions contribute which are not sensitive to the fluctuations. A natural source of large σ 's are configurations of larger than average transverse size. One can expect that the gluon field is enhanced in these configurations while the distribution in x – the light-cone fraction carried by partons of the projectile – is softer for large x leading to a correlation between the distribution over N and distribution over x of a hard collision.

Matching the number of wounded nucleons to the physical observables is certainly a challenging problem in view of fluctuations

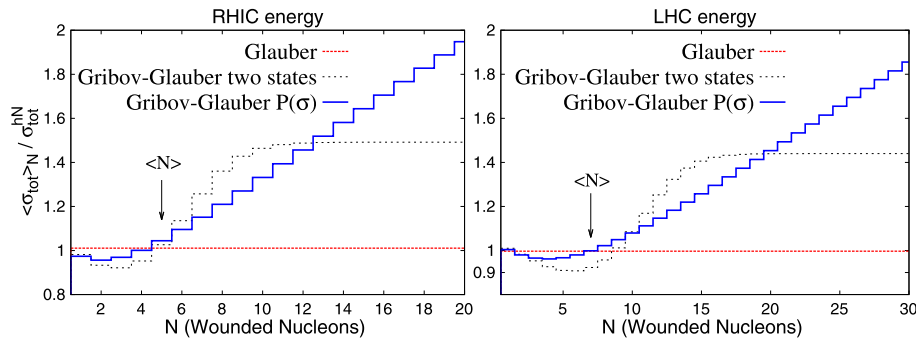


Fig. 4. Effect of fluctuations on the event-by-event fluctuating values of σ_{tot} , for RHC and LHC energies.

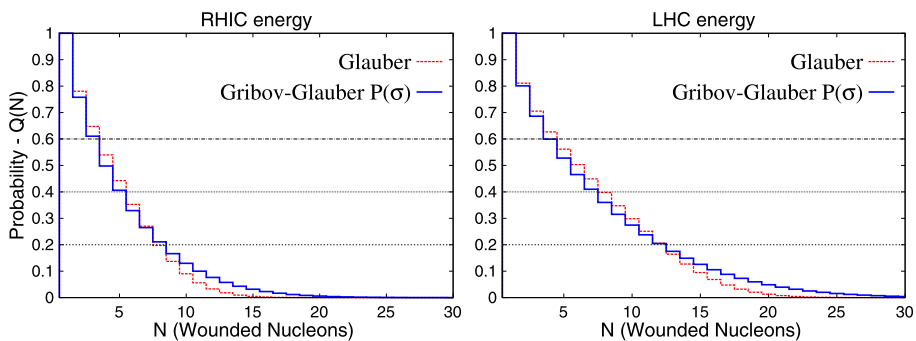


Fig. 5. Fraction of inelastic cross section plotted as a distribution over impact parameter as defined in Eq. (23). Horizontal lines at 0.2, 0.4 and 0.6 correspond to the experimental definition of 20%, 40% and 60% centrality, respectively.

of the impact parameter in the collisions. A model independent treatment of this problem would require a study of pA collisions for different nuclei. Still the central multiplicity appears to be a good observable even in the presence of the color fluctuations. Indeed in the soft interaction dynamics the hadron multiplicity for central rapidities, $y_{c.m.} \sim 0$, does not depend on σ_{tot}^{hN} , as it is determined by the density of partons in a single Pomeron ladder. Hence the hadron multiplicity for $y_{c.m.} \sim 0$ should be about the same for different fluctuations. Also the first studies of the pA collisions at the LHC indicate that to a good approximation the hadron multiplicity for $p_T \geq 1$ GeV is proportional to the number of wounded nucleons calculated in the Glauber model [17]. Hence we expect that selecting events with the $y_{c.m.} \sim 0$ hadron multiplicities: $M/\langle M \rangle \geq 2.5$ should select configurations in the projectile significantly larger than average ones (cf. Fig. 4 right) with significantly different parton distributions.

Correspondingly, a trigger for configurations of smaller than average size would lead to a more narrow distribution in N . One such possibility is to select as a trigger a hard process in which a parton of the proton with $x_p > 0.6$ is involved. One may expect that in this case one selects quark-gluon configurations without $q\bar{q}$ pairs and significantly screened gluon field, leading to σ_{in} significantly smaller than average and hence a strong suppression of large N tail [18]. Such measurements appear to be feasible using the data collected in the 2013 pA run at the LHC in which a significant number of events with large x_p should have been collected. Since this kinematics (for the current LHC detectors) corresponds to very large p_T 's of the jets, one expects that for the inclusive cross section impulse approximation would work very well. Hence it would be possible to avoid issues of the final/initial state interactions and nuclear shadowing in interpreting these data.

A convenient quantity to study these effects experimentally would be a measurement of the distribution over x_p for different

classes of hard collisions at fixed x_A normalized to the distribution in the inclusive pA scattering. A large effect is expected for the central collisions where the hard cross section should be suppressed for large $x_p \geq 0.2-0.3$ and enhanced for $x \leq 0.05$.

Note that such a measurement among other things would allow to test in an unambiguous way the explanation of the EMC effect at large x as due to the dominance of the smaller than average size configurations in nucleon at $x \geq 0.6$; for a recent review see Ref. [19].

We also investigated the impact of fluctuations of the definition of centrality classes. We followed the experimental definition, in which the centrality is proportional to the fraction of total inelastic cross section provided by a given type of events. We can extract from the MC results of Fig. 1 the probability Q_N of having at least N inelastic interactions, irrespective of the impact parameter b (cf. Eq. (7)):

$$Q_N = \frac{\sum_{M=N}^A \int db P_M(b)}{\sum_{M=1}^A \int db P_M(b)}, \quad (23)$$

in such a way that $Q_{N=1} = 1$ by definition. This allows to estimate the fraction of σ_{in}^{hA} arising from a given interval in the number of wounded nucleons. Then, one can choose a centrality class and select the interval in number of wounded nucleons which contributes to that class. In Fig. 5, we have chosen the classes of the 20% most central events by requiring it to provide 20% of the total inelastic cross section and, similarly, we have singled out the 20%–40% and 40%–60% centrality classes, and the 40% most peripheral events as the last class. We use the number of the wounded nucleons corresponding to (closer to) these cuts as limits in N entering in Eq. (24), for the calculation of the curves in Fig. 6. In Fig. 6 we show, for the selected classes, the distribution of events as a function of impact parameter by plotting

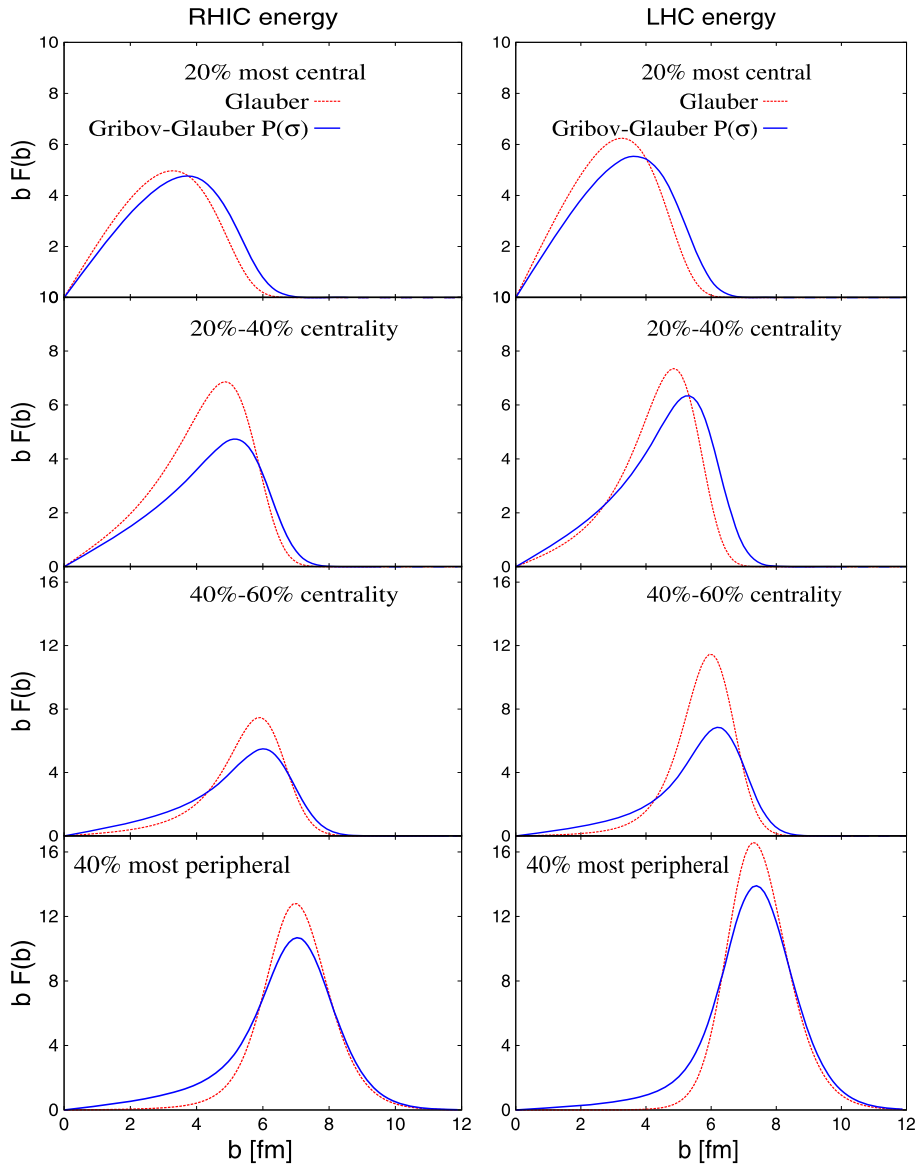


Fig. 6. The distribution over impact parameter, calculated with our MC, of the different centrality classes 20% most central (first row), 20%–40% (second row), 40%–60% (third row), 40% most peripheral (last row), both for RHIC (left) and LHC (right) energies. Red: Glauber result; blue: Gribov-Glauber color fluctuations with $P(\sigma)$ distribution. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

$$bF(b) = b \sum_{N=N_{min}}^{N_{max}} P_N(b), \quad (24)$$

where N_{min} and N_{max} are the values singled out by the cuts described above and shown in Fig. 5. The distribution of Fig. 6 was calculated both with the usual Glauber approach, *i.e.* with a fixed σ_{tot}^{hN} , and with the inclusion of a fluctuating cross section according to $P_h(\sigma_{tot})$. For all the classes and both the RHIC and LHC energies, it can be seen that the fluctuating cross section tends to push the distribution toward larger values of the impact parameter.

5. Conclusions

We have demonstrated that color fluctuations lead to a significant modification of the distribution over the number of nucleons involved in inelastic proton–nucleus collisions at collider energies. Study of the correlations between the soft central multiplicity and

the rate of hard parton–parton interactions in the pA collisions at the LHC would provide a new avenue for investigating the three-dimensional structure of proton. In particular such measurement will allow to test a conjecture that quark–gluon configurations in the proton containing large x_p partons have a significantly smaller than average size.

Acknowledgements

We thank L. Frankfurt and V. Guzey for useful discussions during the preparation of the Letter.

References

- [1] V.N. Gribov, B.L. Ioffe, I.Y. Pomeranchuk, Sov. J. Nucl. Phys. 2 (1966) 549, Yad. Fiz. 2 (1965) 768.
- [2] S. Mandelstam, Nuovo Cim. 30 (1963) 1148.
- [3] V.N. Gribov, Sov. Phys. JETP 26 (1968) 414, Zh. Eksp. Teor. Fiz. 53 (1967) 654.

- [4] V.N. Gribov, *Sov. Phys. JETP* 29 (1969) 483, *Zh. Eksp. Teor. Fiz.* 56 (1969) 892.
- [5] G. Baym, L.L. Frankfurt, M. Strikman, *Nucl. Phys. A* 566 (1994) 149C.
- [6] G. Baym, B. Blattel, L.L. Frankfurt, H. Heiselberg, M. Strikman, *Phys. Rev. C* 52 (1995) 1604.
- [7] M.L. Good, W.D. Walker, *Phys. Rev.* 120 (1960) 1857.
- [8] E.L. Feinberg, I.Ya. Pomeranchuk, *Suppl. Nuovo Cim.* 3 (1956) 652.
- [9] B. Blaettel, G. Baym, L.L. Frankfurt, H. Heiselberg, M. Strikman, *Phys. Rev. D* 47 (1993) 2761.
- [10] B.Z. Kopeliovich, L.I. Lapidus, *Pisma Zh. Eksp. Teor. Fiz.* 28 (1978) 664.
- [11] H.I. Miettinen, J. Pumplin, *Phys. Rev. D* 18 (1978) 1696.
- [12] M. Alvioli, C. Ciofi degli Atti, I. Marchino, V. Palli, H. Morita, *Phys. Rev. C* 78 (2008) 031601.
- [13] M. Alvioli, C. Ciofi degli Atti, B.Z. Kopeliovich, I.K. Potashnikova, I. Schmidt, *Phys. Rev. C* 81 (2010) 025204.
- [14] M. Alvioli, H.-J. Drescher, M. Strikman, *Phys. Lett. B* 680 (2009) 225.
- [15] L. Bertocchi, D. Treleani, *J. Phys. G* 3 (1977) 147.
- [16] V. Guzey, M. Strikman, *Phys. Lett. B* 633 (2006) 245.
- [17] B. Abelev, et al., ALICE Collaboration, arXiv:1210.4520 [nucl-ex].
- [18] L.L. Frankfurt, M.I. Strikman, *Nucl. Phys. B* 250 (1985) 143.
- [19] L. Frankfurt, M. Strikman, *Int. J. Mod. Phys. E* 21 (2012) 1230002.